Nested Bases for Rational PH-Curves

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A curve with rational parametric equations is called "Pythagorean hodograph" or "PH" if its normalized derivative vector is rational as well. PH-curves are useful in situations where not only a geometric point locus but also its time-dependent trajectory need to be modeled. Examples include motion control, animation, or path planning.

Polynomial PH-curves have been studied extensively for more than thirty years. There is an elegant direct approach for their computation by integration of certain vectorial quaternion polynomials. Rational PH-curves are usually computed as envelopes of their osculating planes. While this yields a closed formula for all rational PH-curves, it is difficult to control their degree as unexpected cancellations often occur. The reason for these cancellations is hidden in some convoluted polynomial algebra and is not yet fully understood.

We present an alternative method for computing rational and polynomial PH-curves. Given the curve?s tangent indicatrix and its denominator polynomial, it requires solving a modestly sized system of linear equations. Rather surprisingly, it turns out that polynomial PH-curves are generic in this context and rational PH-curves occur only in special cases. With our approach, PH-curves of bounded degree naturally form an infinite sequence of nested vector spaces for which bases can be computed. This not only gives new insight into the structure of polynomial and rational PH-curves. It also suggested a very clear standardized representation via finite Laurent series and provides computational advantages for applications.